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Predictive Missile Guidance

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Guidance to intercept a maneuvering target is considered. Uncertainty in the predicted target position at interception is handled by constructing a Gaussian-sum approximation to the probability density function of that position. A guidance framework inspired by model predictive control is presented, with the objective of maximizing the probability of successful interception, and simplifications to make the computations feasible are suggested. The scheme is extended to the multitarget case, incorporating uncertain information on target identity. The multitarget procedure allows compromise between progress toward the selected target and keeping as many targets reachable as possible. Simulation results illustrating performance improvements over conventional guidance are presented.

Nomenclature

a	= lateral acceleration
C	= maximum number of maneuver changes allowed
$\mathcal{F}$	= missile effectiveness function
J	= cost function
k	= discrete-time instant
L	= prediction horizon
M	= control horizon, number of update intervals to interception
$\boldsymbol{M}_i$	= maneuver history
N	= number of possible target maneuver actions
$\mathcal{N}(\boldsymbol{x}, P)$	= Gaussian probability density function with mean $\boldsymbol{x}$, covariance P
P	= covariance of state-estimate error
$p(\cdot)$	= probability density function
Q	= covariance of state forcing
$\mathcal{R}$	= set of reachable objects (potential targets)
r	= range to target
r_e	= radius of effectiveness of missile
t_{go}	= time to intercept
$\mathcal{U}$	= set of admissible control values (accelerations)
u	= missile acceleration
$\boldsymbol{x}$	= state vector (with circumflex if estimate)
x, y	= Cartesian position coordinates
$\mu_{i,k+j}$	= maneuver model i applied over period $k+j-1$ to $k+j$
ω_i	= target turn rate i

Subscripts

m	= missile
t	= target
$\mathcal{T}_j$	= target type j

I. Introduction

IN conventional linear-quadratic-Gaussian (LQG) optimal guidance of a missile approaching a target, the computation of the control (missile acceleration) sequence is based on minimum-covariance, unbiased estimates of target state. Joint optimality of the estimator and control, treating the estimated state as if exact (certainty-equivalence control[1]), relies on linearity of the motion model and the observation process. Two factors make this approach difficult in practice: the target-motion model during maneuvers is usually nonlinear in the preferred coordinate system,[2,3] and so linearization errors are incurred, and the target's future maneuvering is uncertain. This paper proposes guidance based on predicting the probability density function (PDF) of target position at interception, for a range of possible maneuvers, by use of nonlinear models. In principle, the PDF should be computed at each update, allowing a minimum expected cost control sequence to be found without the restrictive assumptions of LQG control. In practice, approximations and simplifications will be necessary to reduce the computing load of such a predictive control scheme.

Predictive control [often called model predictive control (MPC)] employs a receding horizon.[4–6] At time k, an updated model of the plant being controlled relates the control-input sequence to the predicted output up to a prediction horizon M steps ahead, with M fixed. To minimize a cost function in the predicted output error and the control, the control-input sequence is optimized (over its admissible range) up to the control horizon $k + L$ and assumed to be kept constant from then on. The first sample of the optimal sequence is applied from time k up to the next observation instant, time $k + 1$. The whole process is then repeated. The guidance law is, thus, open-loop feedback-optimal.[7] Given enough computing power, the optimization need not rely on stringent assumptions to ease analytical solution, and so MPC has become popular in process control, where its ability to handle constraints is valuable and plant dynamics are typically slow.

The missile guidance problem has acceleration set point as control input and, in the version considered here, miss distance as output error. As an MPC problem, it has two nonstandard features: 1) The prediction horizon is the interception time, which does not recede steadily but varies with the assumed future maneuvers and with the accuracy of the predicted missile and target states. 2) It must account for possible future maneuvers and predict the PDF of target position, not just its mean.

This paper considers MPC-style guidance with limited computing power. Section II discusses prediction of the time to intercept and the PDF of target state at that time; the prediction is difficult and guidance accuracy is sensitive to error in it. Simplifying assumptions to keep the prediction load acceptable are proposed. The guidance law is considered in Sec. III, and Sec. IV gives simulation results comparing the predictive control scheme with LQ guidance.

The guidance scheme can be extended to multiple targets, balancing the probability of successfully intercepting the chosen target against that of selecting the correct target. Section V discusses an optimal multitarget guidance scheme, modified in Sec. VI to reduce computing load. Finally, Sec. VII presents multitarget simulation results, illustrating the balance between delaying target selection for correct identification and maximizing the probability of reaching the selected target.

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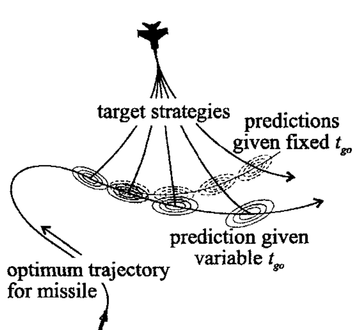
II. Calculation of Time to Intercept and PDF of Predicted Target State

The difficulties in predicting target position at interception stem from uncertainty in interception time. First, the uncertainty renders the PDF of state at interception non-Gaussian; the differences in missile flight time to different interception points alter the times for which the probability density at those points is required. This is so even if the target does not maneuver, the motion and observation models are linear, and the initial state error, process noise (accelerations), and observation noise are all Gaussian.

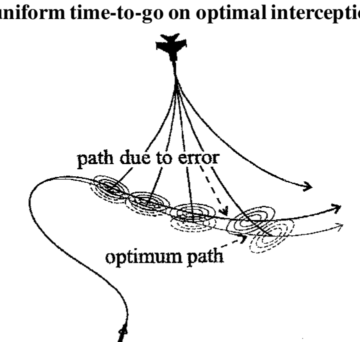
Second, if the target may maneuver, the missile trajectory has to run through a range of maneuver-dependent interception positions, over a range of times. The optimal missile trajectory depends on the PDF at interception, but the PDF depends on the interception times determined by the trajectory. Prediction of the target-position PDF at interception and optimization of the missile trajectory are, thus, closely interwoven problems with an implicit joint solution.

Third, the miss distance is sensitive to error in interception time because of the relative geometry of the optimal trajectory for the missile and the predicted maneuver-dependent target position at intercept. The trajectory runs roughly through the range of possible mean positions resulting from the range of possible maneuvers. For example, if the maneuver is a cross-track acceleration, the trajectory runs across almost at right angles to each possible target track. Miss distance is, therefore, sensitive to any along-track error in final target position, yet this is precisely the sort of error induced by uncertainty in time to intercept. Figure 1 illustrates the difficulty.

Joint iterative refinement of the target-position PDF at interception and the optimal missile trajectory, to an accuracy much better than the maximum permissible miss distance, thus seems necessary at every update, a prohibitive task. However, a crucial assumption allowing an acceptable suboptimal solution is that at long ranges the missile need only travel in approximately the right direction. Hence, at long range the time to intercept need be found only roughly. As interception approaches, the need for accuracy increases but the range of possible conditions at interception narrows, reducing the scope of the computation for more precise guidance. In practice, factors such as increased influence of glint, countermeasures, and removal of necessity to consider more than one target may have a large effect as range falls, but require detailed, scenario-specific consideration beyond the scope of this paper. Also, long-range considerations such as energy conservation and missile maneuvering to improve observability are not included in what follows.



Effect of non-uniform time-to-go on optimal interception trajectory



Effect of error in t_{go} on missile trajectory

Fig. 1 Optimal missile trajectory through PDF of predicted target state.

The long-range guidance strategy to minimize time to intercept would require the missile trajectory to turn at maximum rate followed by straight flight.[8,9] Time to intercept t_{go} could then be estimated by

$$t_{go} = -r/\dot{r} \tag{1}$$

which becomes exact for a constant-velocity target once the turn is completed. The guidance scheme presented here employs this estimate at all times and for all target trajectories. Furthermore, the variation in t_{go} with interception position over the main peak of target-position PDF is neglected. The justification is the reduced extent of the peak as the missile closes, because of improving observation accuracy (particularly cross heading) in the absence of glint and shrinking time for further target maneuver.

Simplification is also necessary to reduce the heavy computing implied by dependence of the PDF of predicted target state at interception on maneuver history. Consider a maneuver history well approximated by a sequence

$$\boldsymbol{M}_i \equiv \left\{\mu_{j1(i),k+1}, \mu_{j2(i),k+2}, \ldots, \mu_{jM(i),k+M}\right\} \tag{2}$$

over the M update intervals up to interception. In each interval, the target motion is described by one of N motion models μ_{jM}, $j = 1, 2, \ldots, N$, with an associated probability. With the target able to choose between N actions over each interval, and if the PDF is an N-component Gaussian sum (as in multiple-model trackers[2,3,10]), then N^{M+1} predictions must be made at every update, an excessive number unless M is very small. The number of maneuver sequences can be reduced by excluding those with low probability (e.g., sustained high-rate turns or very rapid turn reversals) and/or by restricting the number of motion changes. Even so, the number may be large. If up to C changes are allowed, there are

$$S_C = \sum_{c=0}^{C} \frac{(M-1)!}{c!(M-1-c)!} N(N-1)^c \tag{3}$$

maneuver sequences; for example, with $M = 10$ and $N = 5$, $S_1 = 185$, $S_2 = 3065$, and $S_3 = 29{,}945$. The question, therefore, arises whether guidance can be based on less than the complete PDF of predicted target position, recognizing that sensitivity to error in interception time and interdependence of that time and the missile control sequence will in any case prevent high precision. A suboptimal objective, conforming with normal MPC practice, is to reach the mean predicted target position without trying to optimize the missile path through the PDF for successful interception. This reduced aim still takes into account the range of possible maneuvers and the non-Gaussian nature of the PDF.

Use of the mean predicted position permits a further simplification. It can be shown (Ref. 11, Appendix 9) that, as the missile closes with the target, the mean position at intercept assuming no future changes in maneuver (i.e., over all N current motion possibilities only: $C = 0$) tends to the mean over all maneuver strategies. This results from convergence to zero of linearization error in turn-rate-dependent terms in the state equations as time to go t_{go} tends to zero. Although error is incurred for finite t_{go}, the influence of nonlinearities in the motion models is retained, with increasing accuracy as t_{go} falls, ensuring improvement over LQ guidance.

For the reduced set of fixed maneuver sequences $\boldsymbol{M}_i$, $i = 1, 2, \ldots, N$, the prediction of target-state PDF at interception is

$$p(\hat{\boldsymbol{x}}_{k+M|k}) = \sum_{i=1}^{N} p(\hat{\boldsymbol{x}}_{k+M|k} \mid \boldsymbol{M}_i) Pr(\boldsymbol{M}_i) \tag{4}$$

where for multiple-model trackers the PDF of current target state is the Gaussian sum

$$p(\hat{\boldsymbol{x}}_{k|k}) = \sum_{i=1}^{N} \mathcal{N}\left(\hat{\boldsymbol{x}}_{k|k}^{\mu_{i,k}}, P_{k|k}^{\mu_{i,k}}\right) Pr(\mu_{i,k}) \tag{5}$$

Minimum-covariance, unbiased single-step, M-sample-interval predictions[12] are of the form, to first order,

$$\hat{x}^{\mu_{i,k}}_{k+M\mid k} = f\left(\hat{x}^{\mu_{i,k}}_{k\mid k}, M_i, M\right)$$

$$P^{\mu_{i,k}}_{k+m\mid k} = F(M_i) P^{\mu_{i,k}}_{k\mid k} F(M_i)^T + Q \tag{6}$$

where $F(M_i)$ is the Jacobian with respect to $\hat{x}^{\mu_{i,k}}_{k\mid k}$ of $f(\hat{x}^{\mu_{i,k}}_{k\mid k}, M_i, M)$ at M_i. With $C = 0$, only N predictions need be made so that, for example, a constant-speed target traveling in the (x, y) plane with turn rate in the range covered by $\{\omega_1, \omega_2, \ldots, \omega_N\}$ has

$$\hat{x}(k+M) = \hat{x}(k) + \sum_{i=1}^{N}\left[\frac{\sin\omega_i t_{go}}{\omega_i}\hat{x}(k) + \frac{1-\cos\omega_i t_{go}}{\omega_i}\hat{y}(k)\right] Pr(\omega = \omega_i)$$

$$\hat{y}(k+M) = \hat{y}(k) + \sum_{i=1}^{N}\left[\frac{\cos\omega_i t_{go}}{\omega_i}\hat{x}(k) + \frac{\sin\omega_i t_{go}}{\omega_i}\hat{y}(k)\right] Pr(\omega = \omega_i) \tag{7}$$

III. Guidance Law

The prediction process produces an approximate PDF of target state at interception. The guidance law has ideally to maximize the cumulative probability, over the missile trajectory, of successful interception. This would require integration along the future trajectory and with respect to the distance between missile and target, with weighting by missile effectiveness as a function of distance. The PDF of predicted target position has to cover a range of possible interception times and might also have to allow for uncertainty in predicted missile position, due to uncertainty in missile state estimates, control response, and disturbances. If the computing is simplified by optimizing for a single predicted interception instant and omitting uncertainty in future missile behavior (which in any case raises no essentially new issues), the probability of success is

$$J_k = \int \mathcal{F}\{r[x_t(k+M), x_m(k+M)]\} p_k[x_t(k+M)]\, \mathrm{d}x_t(k+M) \tag{8}$$

where missile effectiveness is specified as $\mathcal{F}\{r[x_t(k+M), x_m(k+M)]\}$ and $r[x_t(k+M), x_m(k+M)]$ is the miss distance at time $k+M$. The control sequence computed at time k, with the control and prediction horizons equal, is

$$\{u^*\}_k \equiv \left\{u^*_k, u^*_{k+1}, \ldots, u^*_{k+M-1}\right\} = \arg\max_{\{u\}_k} J_k \tag{9}$$

with the future missile state given by a one-step nonlinear time update

$$x_m(k+M) = g[x_m(k), \{u\}_k, M] \tag{10}$$

Missile effectiveness is specified in Ref. 13 as

$$\mathcal{F}\{r[x_t(k), x_m(k)]\} = \begin{cases} 0.9 & \text{for} \quad r \le r_e \\ 0.9\exp\left[-4(r/r_e - 1)^2\right] & \text{for} \quad r > r_e \end{cases} \tag{11}$$

but the common simple alternative

$$\mathcal{F}\{r[x_t(k), x_m(k)]\} = \begin{cases} 1 & \text{for} \quad r \le r_e \\ 0 & \text{for} \quad r > r_e \end{cases} \tag{12}$$

reduces the optimization to

$$\{u^*\}_k = \arg\max_{\{u^*\}_k \in \mathcal{U}} \int_{r[x_t(k+M), x_m(k+M)] \le r_e} p_k[x_t(k+M)]\, \mathrm{d}x_t(k+M) \tag{13}$$

where $\mathcal{U}$ is the admissible set of control values. The input sequence thus directs the missile through the predicted (fixed-time) target PDF so as to maximize the probability within the radius of effectiveness, as in Fig. 2a.

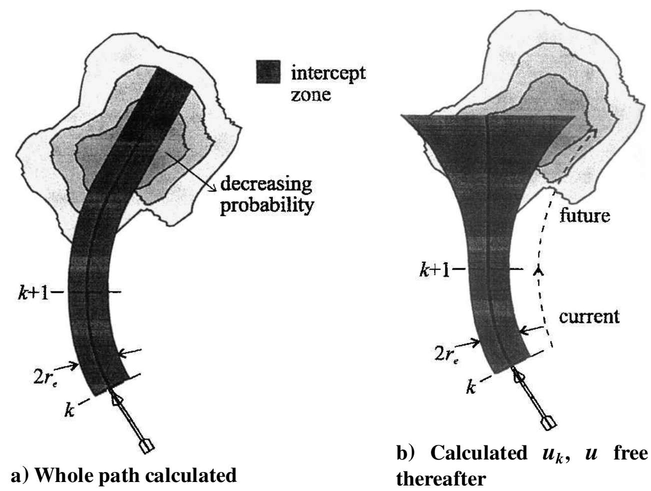


Fig. 2 Alternative approaches to finding optimal missile trajectory.

MPC applies only u^*_k, the first in the sequence, and so there exists a more circumspect alternative, computing for each value of u_k the set of target states reachable within distance r_e, up to time $k+M$, with control u_k and any admissible sequence $\{u_{k+j}, 1 \le j \le M-1\}$, then maximizing with respect to u_k the probability covered by the reachable set, as in Fig. 2b. For a single target, the effective support of the PDF shrinks as t_{go} falls, and these alternatives have similar performance. For multiple targets, they give significantly different results, as will be seen in Sec. VII.

Exact analytical evaluation of the integral in Eq. (13) is prevented by two facts: even if the current missile-state PDF $p(\hat{x}_{k\mid k})$ is a Gaussian sum, the nonlinear prediction to time $k+M$ [approximated by Eq. (6)] renders its components non-Gaussian, and the missile track through the PDF $p_k[x_t(k+M)]$ may not be straight. A numerical technique for Eq. (13) was investigated in Ref. 11, but found too expensive, and so one must consider how to approximate the PDF and the track through it. At long range, the components of predicted target-position PDF produced by different maneuver values are likely to be widespread enough to prevent them from being combined in any good, simple approximation, but in any case refined trajectory optimization is not justified by the quality of the prediction. As range falls, and with it the time remaining for different possible maneuvers to act, the spread decreases and so does interest in the PDF tails. Ultimately, the whole PDF may be represented as a single peak, perhaps approximated as Gaussian.

The optimal trajectory can be expected to pass close to the peak of a Gaussian single-peak approximation to the PDF unless missile track is sharply curved close to interception and the effectiveness radius r_e is small compared with the PDF spread; in either guidance scheme described, the optimal trajectory passes through the peak for any given missile heading. Consider now whether the optimal heading can be found readily. With a Gaussian PDF approximation, an ellipsoidal contour can be found beyond which the probability density is too low to influence optimality signficantly. If r_e is no larger than the shortest half-axis of this ellipsoid, the best straight-line trajectory is clearly along the longest axis. It remains optimal, although no longer uniquely so, if r_e exceeds the shortest half-axis. However, this trajectory maximizes the spread of time to intercept and, hence, the error in treating the PDF as at a fixed time. Moreover, at ranges long enough to permit missile maneuvering, a single-peak Gaussian approximation may well be invalid. For these reasons, the shape of the ellipsoid is likely to be a poor guide to the best trajectory.

Given these difficulties, explicit trajectory optimization is not attempted. The guidance law just turns the missile heading as rapidly as possible, as in minimum-time-to-intercept guidance,[8,9] toward the mean predicted target position at interception, as computed through Eq. (6). In doing so, the non-Gaussian representation of the PDF and the nonlinearity in the motion model are retained.

IV. Simulation Results for Guidance Against Single Target

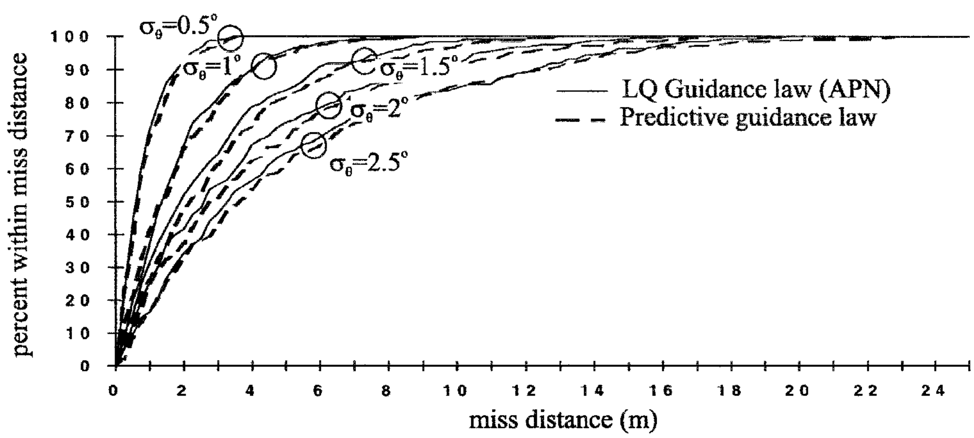
Effectiveness of the guidance law relates to the miss distance, and so most published comparisons of guidance laws employ mean miss distance or percentage of closest approaches within a specified distance. Here, performance will be assessed more informatively by the cumulative distribution of miss distance (between centers of missile and target); other measures may be found from it if required.

The tests use two scenarios. The first has a constant-velocity target, for which LQ guidance is optimal as motion-linearization errors are zero. The target is assumed not to know the missile guidance strategy, but, by behaving randomly, a target can increase tracker errors and, thus, degrade missile performance.[14] In the second scenario, therefore, the target switches at random instants between opposite-sense circular turns.

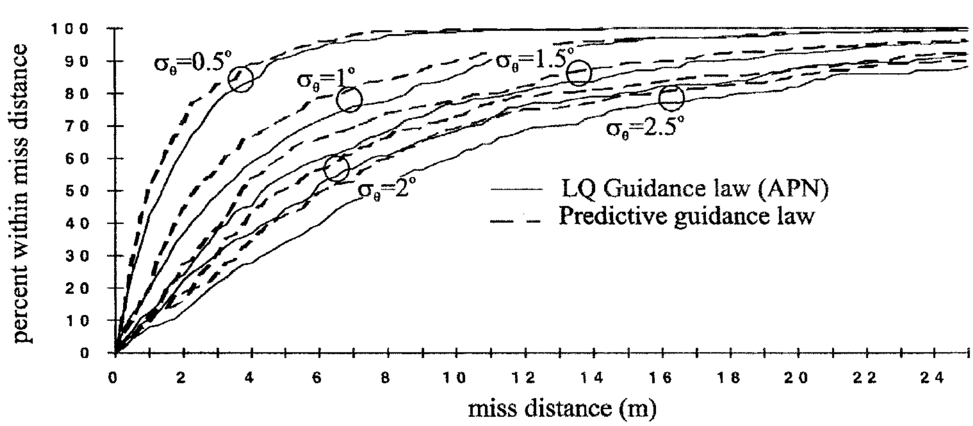
In both tests, the target starts 5 km north and 5 km east of the missile and travels east at 300 m/s. White, zero-mean, uncorrelated Gaussian accelerations of standard deviation 0.1 m/s^2 act across track and along track. The randomly maneuvering target switches between extra cross-track accelerations of ± 80 m/s^2 according to a Poisson process with rate 1/s.

The interacting multiple-model (IMM) tracker[2,3,10] with cross-track acceleration values $a_n = \{-100, -50, 0, 50, 100\}$ m/s^2 receives range and azimuth measurements taken at 20/s for 10 s, with additive, zero-mean, white, Gaussian errors of standard deviation 50 m in range and various values from 0.5 to 2.5 deg in azimuth. The missile processes 10 samples to let the tracking filter settle, then moves at 1200 m/s throughout. Its cross-track acceleration can take any value between ± 300 m/s^2, and for simplicity its dynamics are taken as instantaneous. Augmented proportional navigation, a representative LQ guidance law known to be one of the best LQ techniques for maneuvering targets,[15,16] is used, with navigation constant 3. Monte Carlo trials of 250 runs are performed.

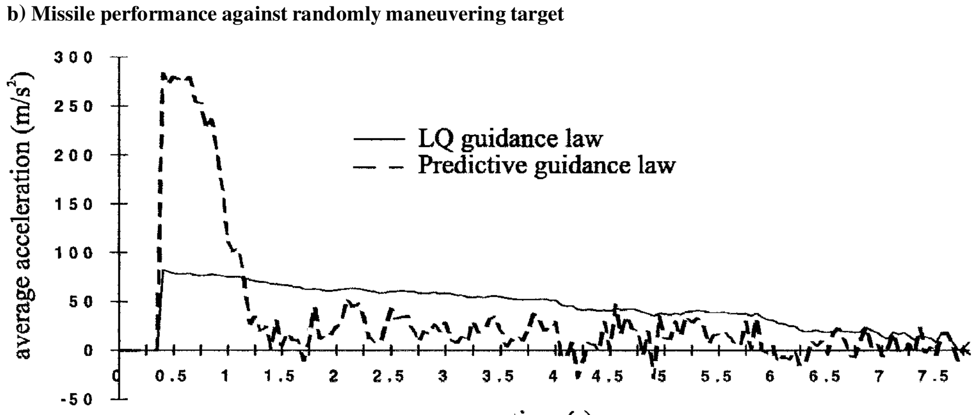
Figure 3 shows the cumulative distributions of miss distance for the constant-velocity and maneuvering target. For both, accuracy declines (the distribution has smaller initial gradient) as azimuth



a) Missile performance against constant-velocity target



b) Missile performance against randomly maneuvering target



c) Comparison of missile acceleration demand

Fig. 3 Simulation results for LQ and predictive guidance.

error and, hence, tracker error increase. With the constant-velocity target (Fig. 3a), the LQ guidance law performs marginally better, being optimal. The difference between LQ and predictive guidance performance changes little with azimuth error. For the maneuvering target (Fig. 3b), predictive guidance does consistently better than LQ, the accuracy of which falls more rapidly as azimuth error increases. This can be explained by the influence of linearization errors. The IMM tracker combines state estimates from a set of linearized models into an optimal estimate. This is then used in a linearized motion model, so that the acceleration is perpendicular to the reference interception course, for the LQ guidance calculation.[1] The predictive scheme incurs the same track linearization errors but predicts with nonlinear models, thus reducing linearization error in the guidance law.

Figure 3c shows the acceleration demand for each guidance scheme. For a navigation constant of 3, the LQ acceleration falls linearly with time,[16] whereas predictive guidance has a large initial acceleration while the missile turns toward the interception heading but low accelerations later. LQ guidance minimizes acceleration demand overall.[1]

V. Optimal Guidance Against Multiple Targets

Next, the predictive guidance idea will be extended to multiple targets, and simulation results will illustrate balancing of the probability of intercepting the selected target against the probability of selecting the correct one.

Although tracking of multiple objects has received substantial attention,[17,18] relatively little has been published on missile guidance against multiple targets.[19] The topic is gaining importance as prospects improve of exploiting collateral information about the identity of each target, derived, for instance, from observation of its dynamics,[20] checking against performance bounds (Ref. 11 and hereafter), interpreting its radar signature, or fusing measurements from dissimilar sensors. Assume that all information at time k on the identity of target i, without regard to what is known of the other targets, is encapsulated by $Pr(i \mid i, k)$. The overall probability that target i of N in total is the intended one is then

$$Pr(i \mid k) = \frac{1}{\alpha} Pr(i \mid i, k) \prod_{\substack{j=1 \\ j \neq i}}^{N} [1 - Pr(j \mid j, k)] \tag{14}$$

These probabilities weight the predicted PDFs of position of all N objects at predicted interception time to produce a PDF $p(\hat{\mathbf{x}}_{k+M \mid k})$ of position of the intended target. The predicted time at interception $k + M(i)$ may well vary from one object to another, requiring the missile-acceleration sequence to maximize the sum, over all possible interception times and correct targets, of the weighted probabilities of intercepting the corresponding objects. Morever, the significance of such differences in time may not decrease as interception approaches. However, to keep the optimization computationally feasible, the same approximation will be made as for a single target, taking the time at interception as unique and independent of the state at that time.

Recall the alternative guidance strategies: 1) find $\{u_k^*, u_{k+1}^*, \ldots, u_{k+M-1}^*\}$ to maximize the total probability within the missile's radius of effectiveness r_e at time $k + M$ or 2) find u_k^* alone to maximize the total probability within r_e of the missile's reachable set at $k + M$. When differing objects give distinct peaks in the predicted intended-target position PDF $p(\hat{\mathbf{x}}_{k+M \mid k})$, strategy 1 aims the missile at or near the peak corresponding to the object most likely to be the intended target. If a rejected target later becomes the most likely, it may no longer be reachable. Strategy 2 hedges against such changes, because it postpones concentration on a narrow strip through the PDF. However, some progress toward the current highest peak is lost, increasing the risk of losing an intended target generating that peak. Figure 4 illustrates how. The control u_k^* attempts to position the reachable set of the missile at interception time so that the probability covered by the set has the same rate of change with u_k in all directions. Early in the engagement (Fig. 4a) the set encompasses two intended-target PDF peaks, allowing two potential targets. Later (Fig. 4b), the boundary of the shrinking reachable set

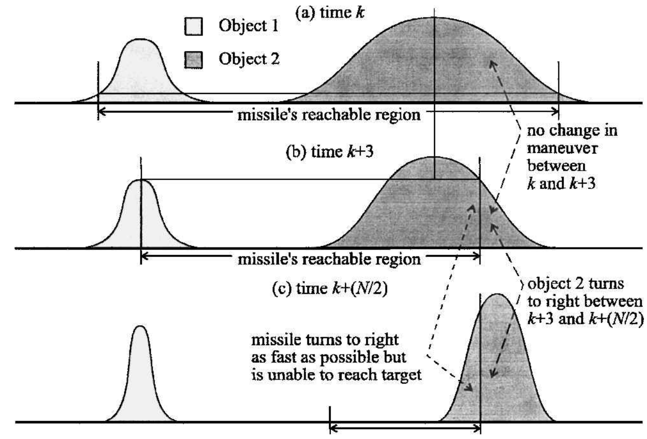


Fig. 4 Possible loss of target when maximizing probability within reachable set.

slips past one of the peaks and shortly that object is rejected. However, by then the control may have been so biased by that peak that a modest maneuver can move the remaining target beyond reach, as in Fig. 4c. Note, incidentally, that this strategy does not require a separate, explicit decision rule for rejecting potential targets.

The two strategies can be viewed as extreme cases of a more general strategy that optimizes the first L control values, with $1 \leq L \leq M$, to maximize the probability covered by the reachable set resulting from those L values. That is, the control is free to take any values in its admissible set for times beyond $k + L - 1$. Strategy 2 has $L = 1$ and strategy 1 has $L = M$. There is a loose analogy with conventional MPC, where the control is assumed to be kept constant beyond the control horizon.[4–6] The choice of L should depend on the information at hand; if the identity of the intended target is clear, L should be large to focus on the highest PDF peak, whereas uncertain identification of the intended target requires small L to keep as large a range of potential targets in reach as possible. All other things being equal, greater maneuver capability should move the choice in the direction of larger L to force earlier target selection.

Computation of the reachable set, followed by maximization of the interception probability by integrating the predicted intended-target position PDF over the reachable part of its support, is far too large a task to be discharged in real time, and so simplification is necessary, as it was for a single target.

VI. Simplified Guidance Against Multiple Targets

The following simplified, suboptimal scheme still makes use of target-identity information and hedges against selection of the wrong target, while conforming with the simplified single-target scheme. The scheme keeps the $Pr(i \mid k)$-weighted PDFs of predicted position at interception for the N objects distinct, instead of merging them into one. When the missile is pursuing a particular target, it employs the same tactic as in Sec. III, maximum-rate turn then straight flight, based on the mean from that target's PDF.

Target selection is straightforward in the simplified scheme. Define an object as effectively reachable if all of its predicted positions at interception with probability density above a specified value lie within the reachable set of the missile. Denote by $\mathcal{R}$ the set of effectively reachable objects. Target selection is, thus, the reduction of $\mathcal{R}$ to a single object. The guidance has two, possibly conflicting, objectives:

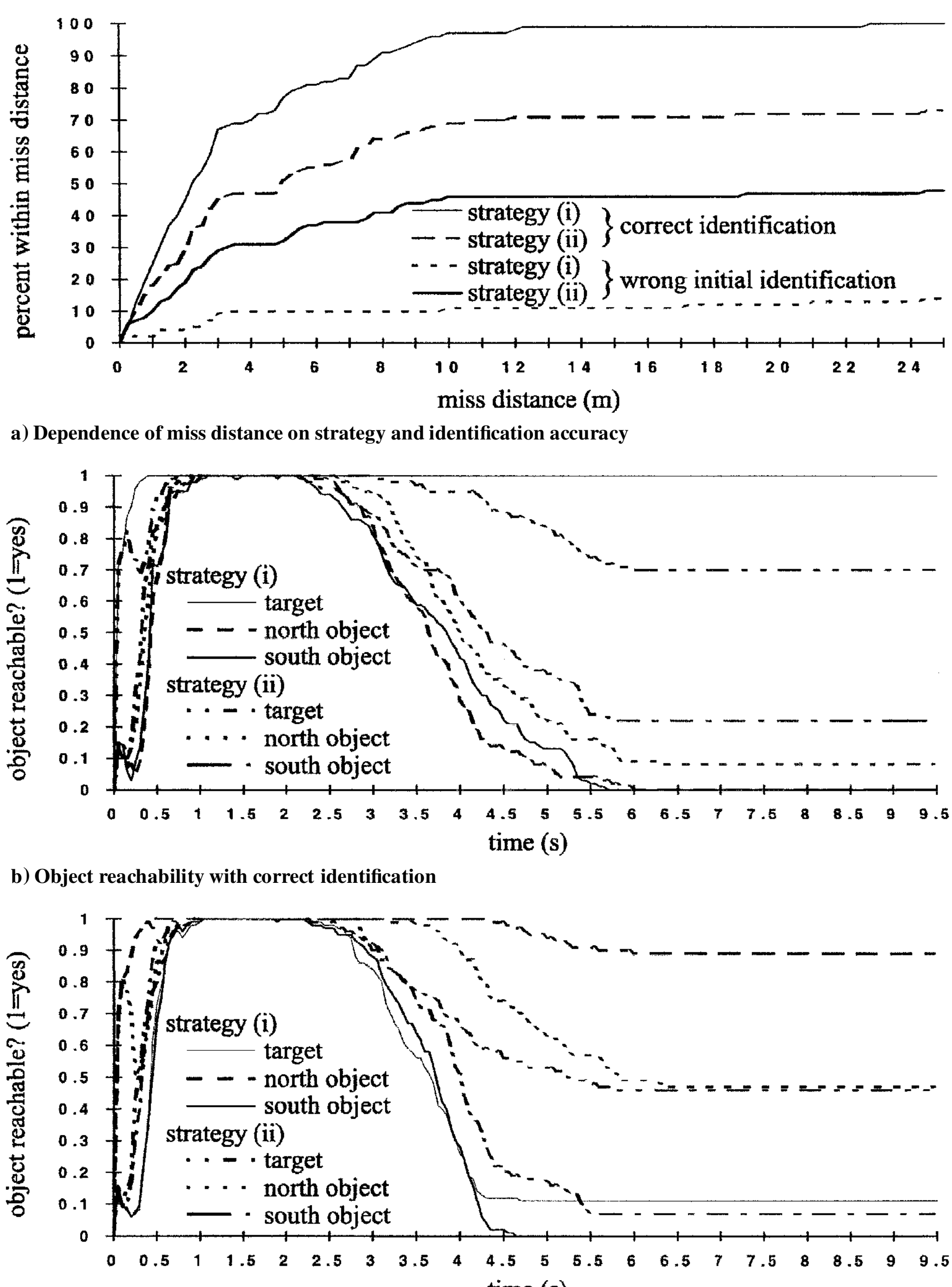
1) Maximize the probability of intercepting the selected target from $\mathcal{R}$, allowing for later maneuvers (roughly strategy 1 given earlier).

2) Maximize the probability that $\mathcal{R}$ includes the intended target (roughly strategy 2).

Objective 1 is achieved by turning onto an interception course as rapidly as possible, maximizing the remaining scope for missile maneuver should the target maneuver later. While all N objects remain effectively reachable, objective 2 is satisfied, allowing the guidance freedom to optimize a subsidiary criterion. Objective 1 suggests that the criterion should be the probability of intercepting the object most likely to be selected, that is, with the largest weighted probability over its effectively reachable region. Eventually the missile reaches a point where continuation of the subsidiary criterion would render an object no longer effectively reachable. Objective 2 then dictates that the guidance should continue along the path that keeps that object effectively reachable, until a point is reached where another object is about to lose effective reachability. A decision as to which object to abandon must then be made: The one with the smaller weighted probability is abandoned. This process continues until only one target is left. Target rejection is now explicit, in contrast to strategy 2, but still simple.

VII. Simulation Results for Guidance Against Multiple Objects

Three objects are considered, because two require only a single decision. All start 5 km north and 5 km east of the missile's initial position. The intended target flies initially east at 300 m/s and the other objects diverge, heading 10 deg north and 10 deg south of east, also at 300 m/s. The target then executes a random sequence of circular turns with accelerations ± 80 m/s^2, switching according to a Poisson process with rate 1/s. As a result, the target track crosses the others. All three objects have additional white, Gaussian, zero-mean cross- and along-track accelerations of standard deviation 0.1 m/s^2. As for the single-target simulations, the IMM tracker uses cross-track acceleration values $a_n = \{-100, -50, 0, 50, 100\}$ m/s^2 and receives range and azimuth measurements at 20/s for 10 s, with additive, zero-mean, white, Gaussian errors of standard deviation 50 m in range and now 1 deg in azimuth. The missile details are as before and the guidance uses $L = 1$ or $L = M$, with effective reachability extending out to the three standard deviation contours of the Gaussian components. (Because the predicted-position PDFs are not Gaussian, one cannot infer the probability of reachability.) Monte Carlo trials of 250 runs are performed.



a) Dependence of miss distance on strategy and identification accuracy

b) Object reachability with correct identification

c) Object reachability with wrong initial identification

Fig. 5 Effect of uncertain target identity on multitarget guidance.

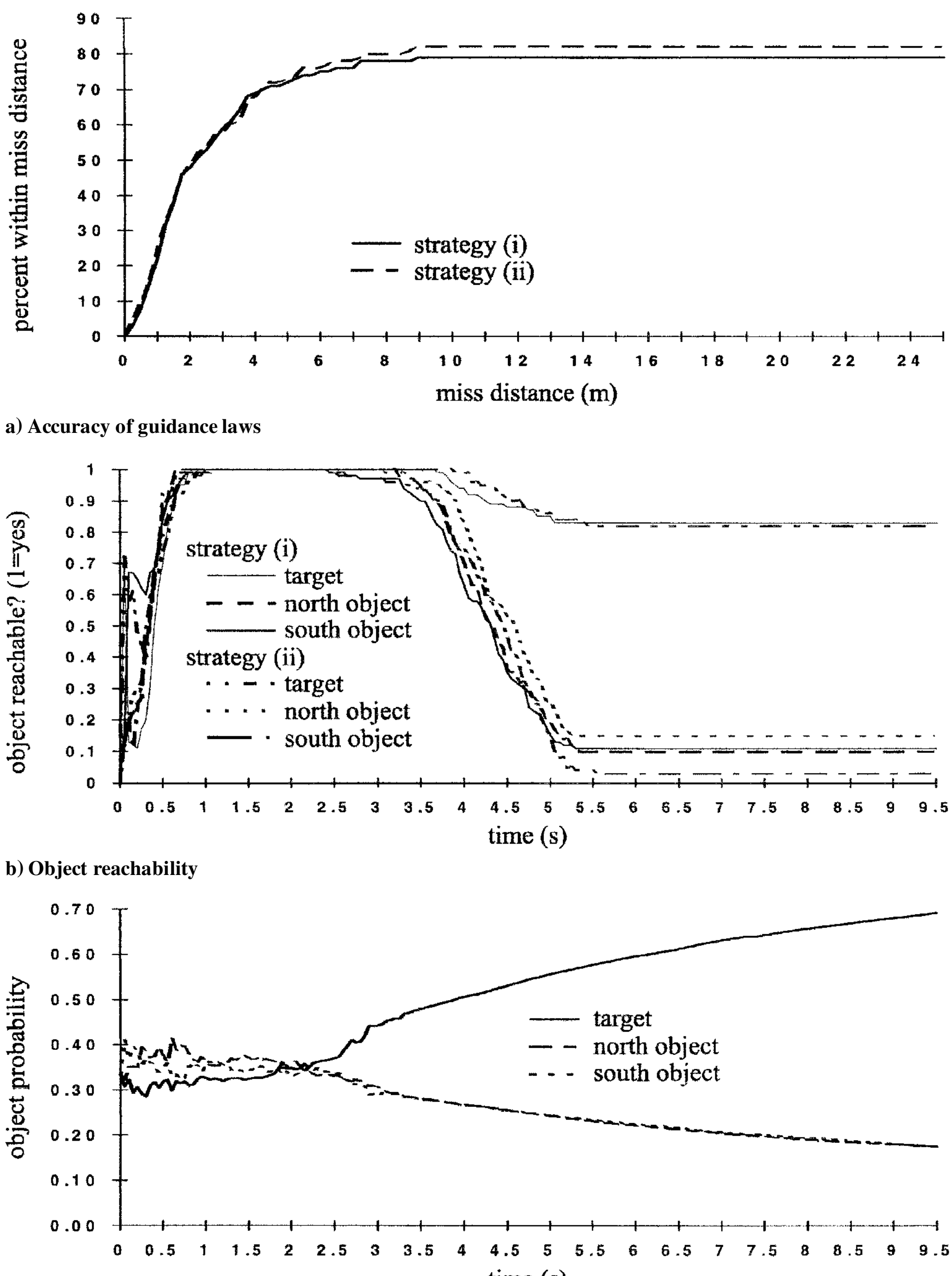
To allow guidance to be studied independently of target identification, identity information is supplied to the missile rather than having to be derived from measurements. Wrong selection can be forced if desired. The target-identity probabilities are simulated as

$$Pr(i \mid k) = \alpha Pr(i \mid k-1) + (1-\alpha)\frac{\Delta Pr(i \mid k)}{\sum_{j=1}^{N} \Delta Pr(j \mid k)} \tag{15}$$

where $\Delta Pr(i \mid k)$ represents information gained at time k about the probability that object i is the intended target, and $\alpha = 0.9$ determines how rapidly the probabilities approach steady state. To simulate correct (on average) identification, $\Delta Pr(i \mid k)$ is a sample from a uniform distribution over [0.3, 0.9] if object i is the intended target and [0.1, 0.7] if it is not. The mean probability of new information classifying the intended target correctly is thus 0.6, that of classifying each other object as the intended target is 0.4, the steady-state probability of correctly identifying the intended target is 0.42, and that of each other object being mistakenly identified as the target is 0.29.

To simulate initially mistaken identification, $\Delta Pr(i \mid k)$ is a random number uniformly distributed in [0.3, 0.9] if object i is the northern object and in [0.1, 0.7] if it is not, until a specified instant from which correct identification is simulated.

The first set of trials covers two situations, one with the correct identification and the other with mistaken identification up to time $k = 80$, each using both guidance objective 1, $L = M$, and objective 2, $L = 1$. With correct identification, objective 1, maximizing the probability of intercepting the selected target, performs much better (Figs. 5a and 5b); under objective 2, the target can escape by maneuvering. However, if the optimization could be performed exactly, strategy 2 rather than objective 2, the difference would be smaller. For instance, if the target were to maneuver so that the missile could not pass closer than two standard deviations from its mean predicted position, the optimal solution might still select the



a) Accuracy of guidance laws

b) Object reachability

c) Probability of objects being true target

Fig. 6 Interception with bounds on speed to identify target.

correct target, with reduced probability of interception, whereas the suboptimal scheme abandons the target as not effectively reachable.

If wrong (on average) identity information is supplied initially, the wrong object is intercepted in 90% of runs for objective 1 (Figs. 5a and 5c). Objective 2 is more successful because the wrong selection is less likely to have been made by the time correct identification starts. Even so, target maneuvers may still mislead the guidance.

The second set of trials employs performance bounds on speed to identify objects. The scenario is as described earlier, but the target cannot exceed 300 m/s and the other objects 290 m/s. The identity information supplied to the missile is only these bounds and the knowledge that there is one target. The probability of object i being of type j is computed as

$$Pr(i \in \mathcal{T}_j) = \frac{1}{\gamma k}\sum_{t=1}^{k} f[\hat{V}_t(i), j] \tag{16}$$

where

$$f[\hat{V}_t(i), j] = \begin{cases} 1 & \text{if } \hat{V}_t(i) \le V_{\max}(j) \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

Ideally, $f[\hat{V}_t(i), j]$ should reflect the uncertainty in state, but the nonlinear relation between Cartesian state and speed makes this impracticable.

Initially, the intended target is identified as the least likely (Fig. 6c) because few of its early speed estimates lie in the range 290–300 m/s restricted to the intended target, but some overestimates for the other objects do lie in that range. As the filter settles, the intended target becomes clearer as more of its speed estimates fall between 290 and 300 m/s, with most for the other objects below 290 m/s. Correct identification thus takes time and, as Figs. 6a and 6b show, the more conservative objective 2, with later decisions, is marginally more successful.

VIII. Conclusions

A conceptual framework has been suggested for guidance of a missile to intercept a target, taking into account uncertainty in the present state and future maneuvers of the target and, in the multitarget case, the identity of each potential target. The framework has much in common with the model predictive control approach popular in process control, but has distinctive features such as a nonreceding prediction horizon and structured disturbances (maneuvers). With multiple targets, the degree of conservatism of the guidance can be controlled by choice of the control horizon up to which the missile acceleration is optimized and beyond which it is only required to be admissible. This determines the compromise between the risk of selecting the wrong target and the loss of progress toward the designated target.

In practice, drastic simplifications are necessary to reduce computing load, including pretending that the time and position of interception are independent and aiming at the mean predicted position of the target, instead of explicitly maximizing the probability of successful interception. Even so, the scheme incorporates non-Gaussian PDFs of predicted position and uncertain, evolving information on target identity. Simulations show that the simplified scheme can improve on conventional guidance. One set of results has illustrated the use of known bounds on speed to aid target identification.

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